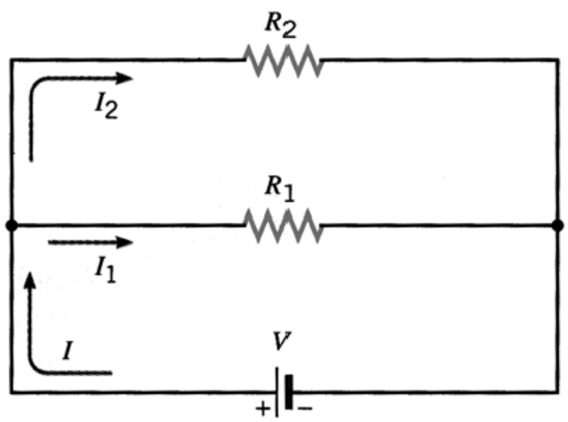
Parallel Circuits

Parallel wiring means that the devices are connected in such a way that the **same voltage is applied across each device**. The diagram below shows two resistors connected in parallel between the terminals of a battery.



Note the following characteristics of devices connected in parallel:

- they are connected along different current pathways
- if the current in one device is interrupted, the current in the other is not

As in a series circuit, it is possible to replace a parallel combination of resistors with an equivalent resistor that results in the same total current and power for a given voltage as the original combination. To determine the equivalent resistance, note that

$$I = I_1 + I_2$$

Where I_1 is the current in resistor R_1, and I_2 is the current in resistor R_2. Since the same voltage V is applied across each resistor, Ohm's law states that:

$$I_1 = \frac{V}{R_1} \text{ and } I_2 = \frac{V}{R_2}$$

Therefore,

$$I = I_1 + I_2$$

$$\frac{V}{R_p} = \frac{V}{R_1} + \frac{V}{R_2}$$

$$V\left(\frac{1}{R_p}\right) = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

where R_p is the equivalent resistance of the parallel circuit.

Thus, two resistors in parallel are equivalent to a single resistor whose resistance is

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

This line of reasoning can be extended to any number of resistors in series, such that:

$$\boxed{\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots}$$

It is interesting to note that the two parallel resistors behave as a single equivalent resistance that is smaller than either R_1 or R_2.

Example 2
A $4.00\ \Omega$ and an $8.00\ \Omega$ resistor are connected in parallel across a $6.00\ V$ battery. Determine

a) the equivalent resistance of the circuit.

b) the total current supplied by the battery.

c) the current in each resistor.

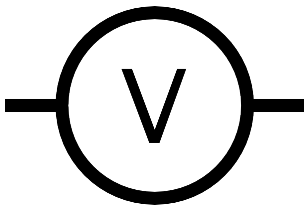
d) the power dissipated in each resistor.

e) the total power delivered by the battery.

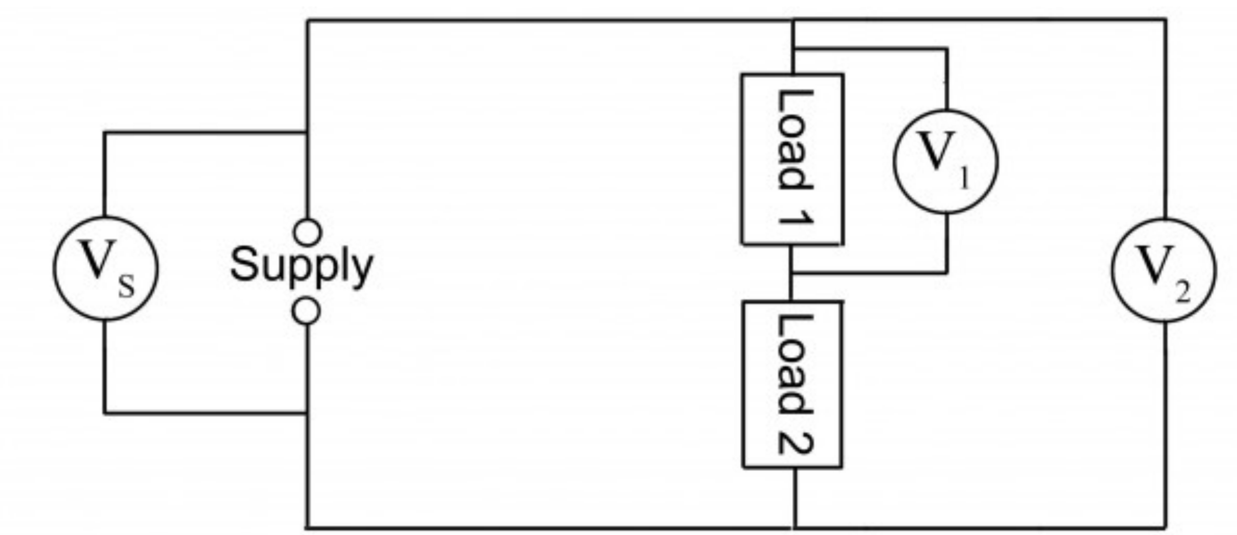
Measuring Voltage

A **voltmeter** is a device used to measure the electric potential difference (voltage) between two points in a circuit. It is typically used to measure the voltage gain or loss across a device. Voltage is measured in Volts (V). The image on the right shows a typical voltmeter.

The symbol used to represent a voltmeter on a schematic diagram is shown below.



In order to measure the voltage increase/decrease across a device the **voltmeter must be connected in parallel** with the device. For example, in the circuit shown below, the voltmeter V_1 is measuring the voltage drop across load 1. The voltmeter V_2 is measuring the voltage drop across both load 1 and load 2. The voltmeter V_S is measuring the voltage gain in the supply (the battery).



As was discussed in the earlier part of the lesson, the total voltage gain (V_S) is equal to the total voltage drop (V_2). It is also worth noting that the voltage drop across load 1 plus the voltage drop across load 2 should add up to V_2.

Circuits Worksheet #6

1. A 16 Ω loudspeaker and an 8 Ω loudspeaker are connected in parallel across the terminals of an amplifier. Assuming the speakers behave as resistors, determine the equivalent resistance of the two speakers. (5.3 Ω)

2. What resistance must be placed in parallel with a 155 Ω resistor to make the equivalent resistance 115 Ω? (445.6 Ω)

3. How many 4 Ω resistors must be connected in parallel to create an equivalent resistance of one-sixteenth of an ohm? (64 resistors)

4. Two resistors, 42 and 64 Ω, are connected in parallel. The current through the 64 Ω resistor is $3\,A$. (a) Determine the current in the other resistor. (b) What is the total power consumed by the two resistors? ($4.57\,A$, $1454\,W$)

5. A coffee cup heater and a lamp are connected in parallel to the same $120\,V$ outlet. Together, they use a total of $84\,W$ of power. The resistance of the heater is $600\,\Omega$. Find the resistance of the lamp. ($240\,\Omega$)